One-dimensional Thermomechanical Model for Additive Manufacturing using Laser-based Powder Bed Fusion

Yksiulotteinen lämpömekaaninen malli lisäävälle valmistamiselle laserperustaisella jauhepetifuusiomenetelmällä

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KESKI-SUOMEN LIITTO

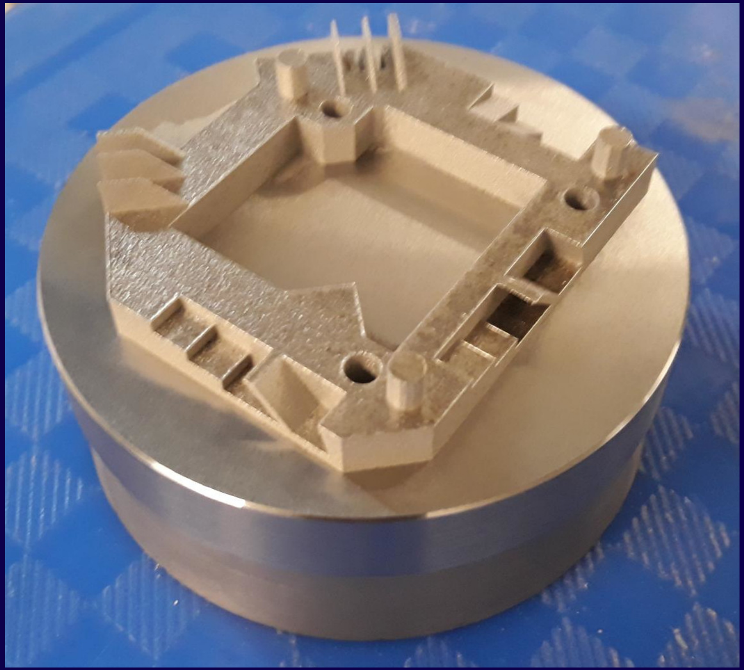
Vipuvoimaa EU:lta 2014–2020

Euroopan unioni
Euroopan aluekehitysrahasto

Lisäävä – Uusien tuotantomenetelmien (3D-tulostus ja ALD) sovellettavuuden kartoitus ja kokeilut keskisuomalaisten pk-yritysten kasvun tukijana. 9.3.2022.

Additive manufacturing for metals – overview

- Relatively new technology (2010s)
- Can manufacture shapes not achievable by other methods
- Based on **melting** or **sintering** (partially melting) the metal
- Several varieties exist, for example:
  - Powder Bed Fusion (PBF) vs. Directed Energy Deposition (DED)
  - Laser based vs. electron beam based
- See the book by Milewski (2017).
- JAMK has a *Trumpf TruPrint 1000* laser-based powder bed fusion (L-PBF) additive production system (i.e. 3D printer for metals).



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Laser-based Powder Bed Fusion (L-PBF)

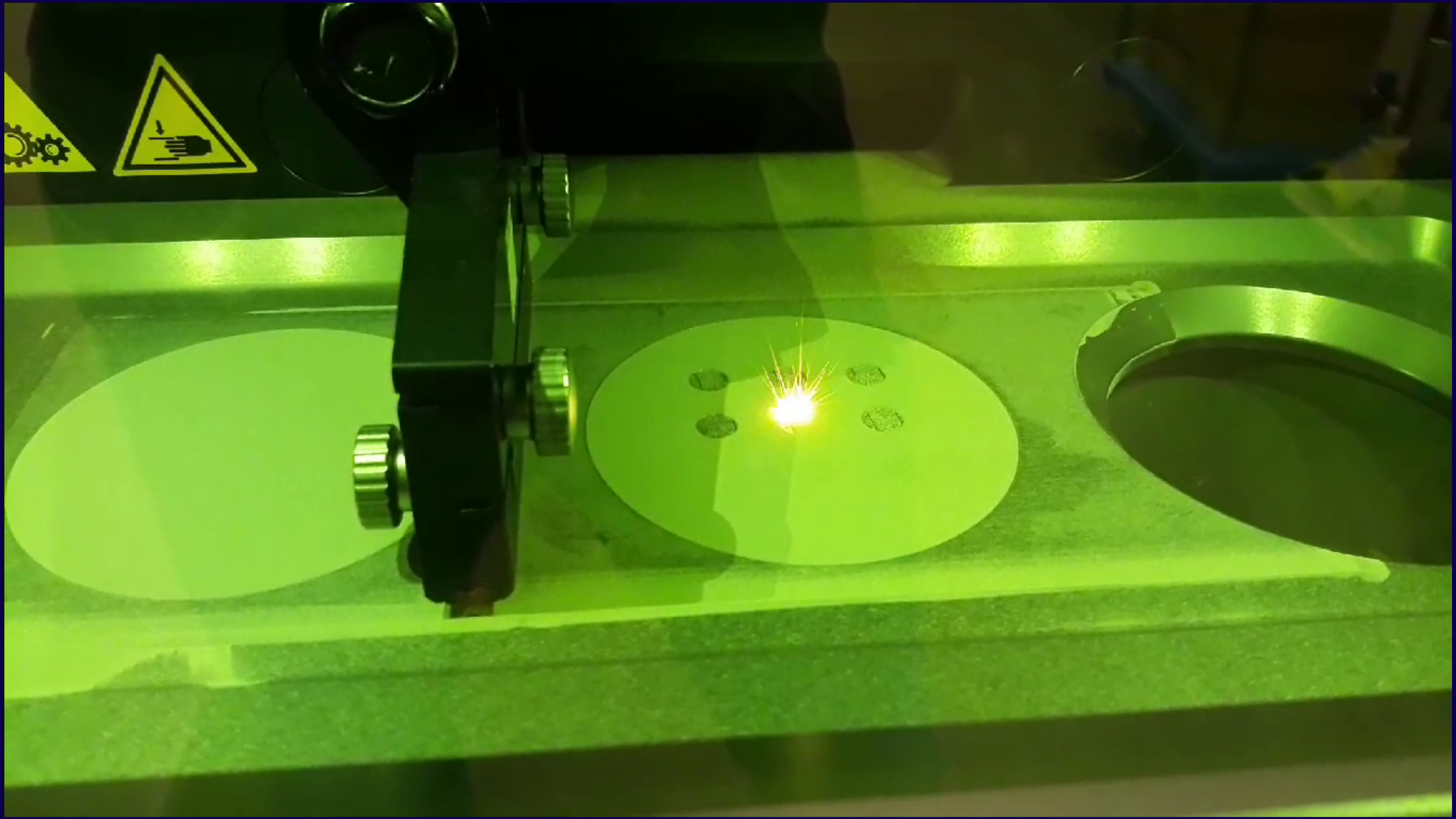


Photo courtesy of Apricon / Trumpf.

Laser-based Powder Bed Fusion (L-PBF)

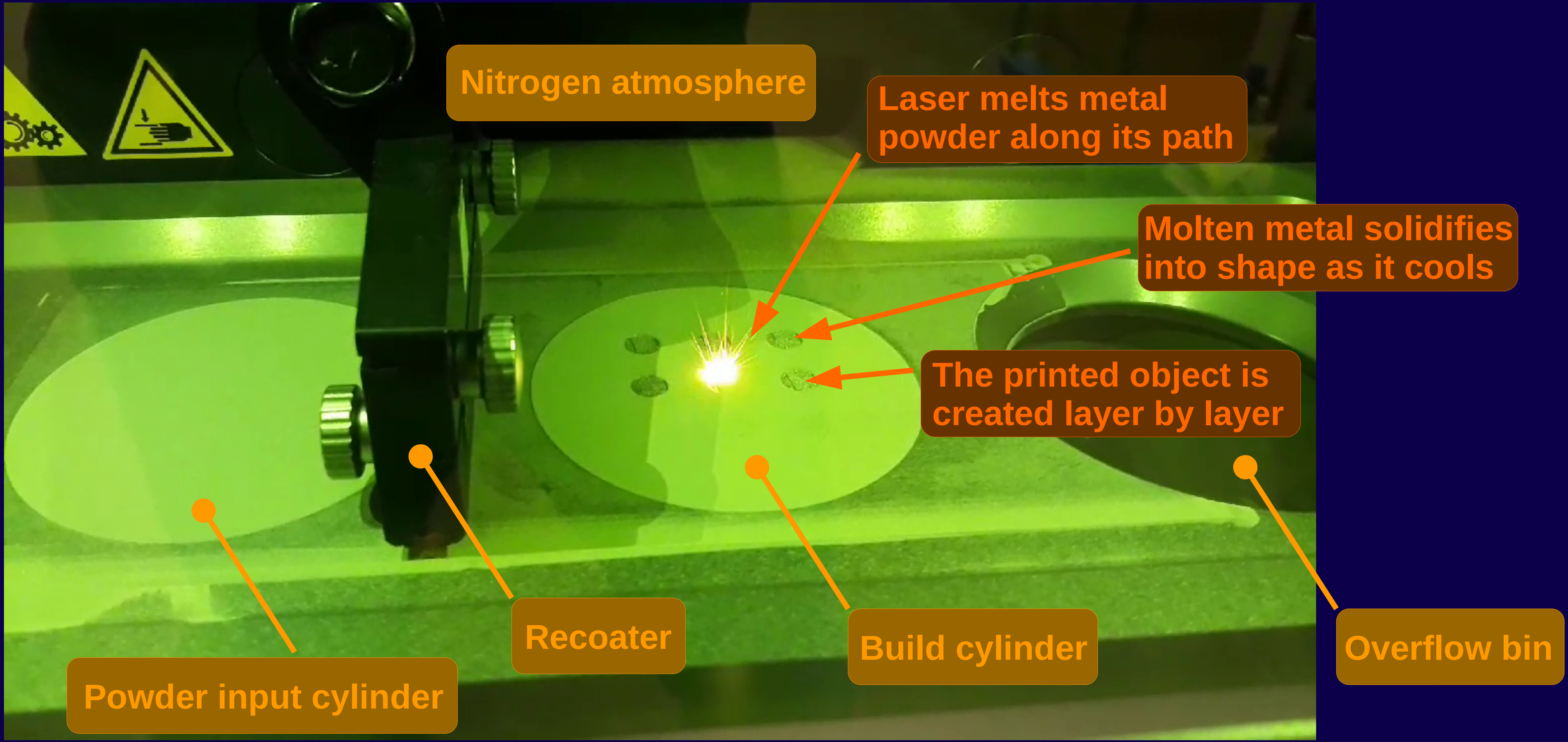
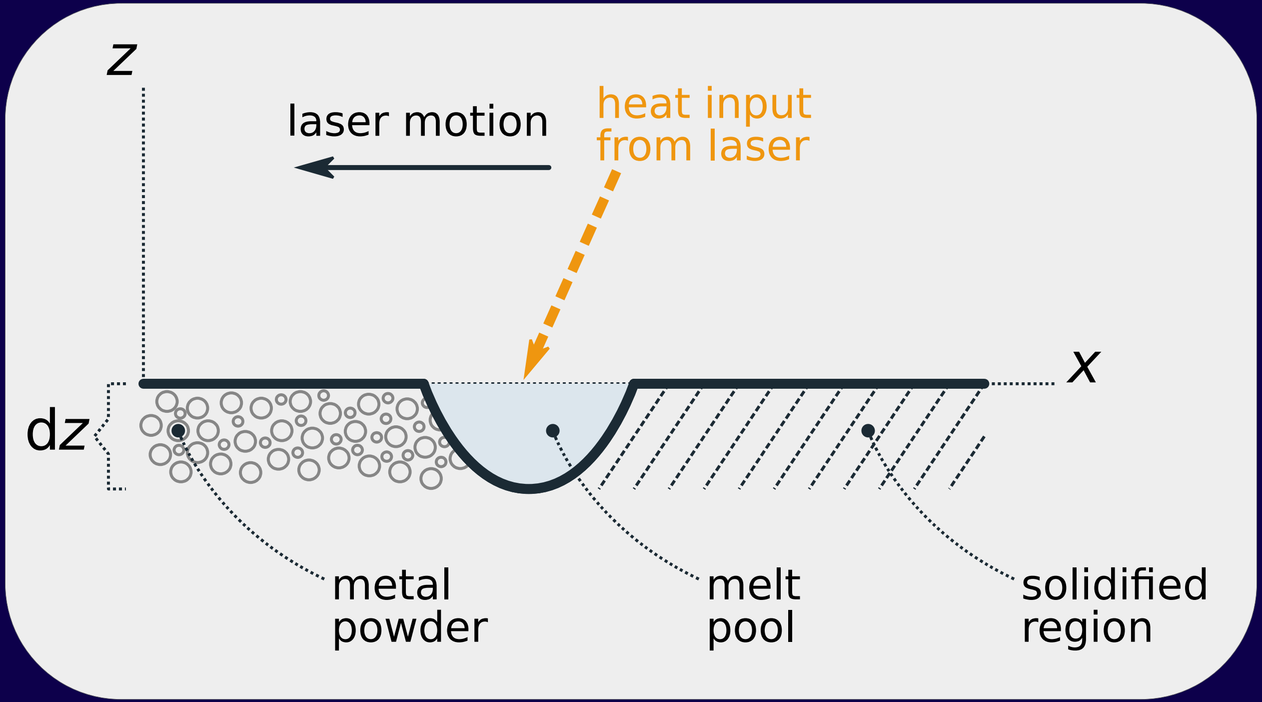


Photo courtesy of Apricon / Trumpf.

Consider printing metal in a straight line...

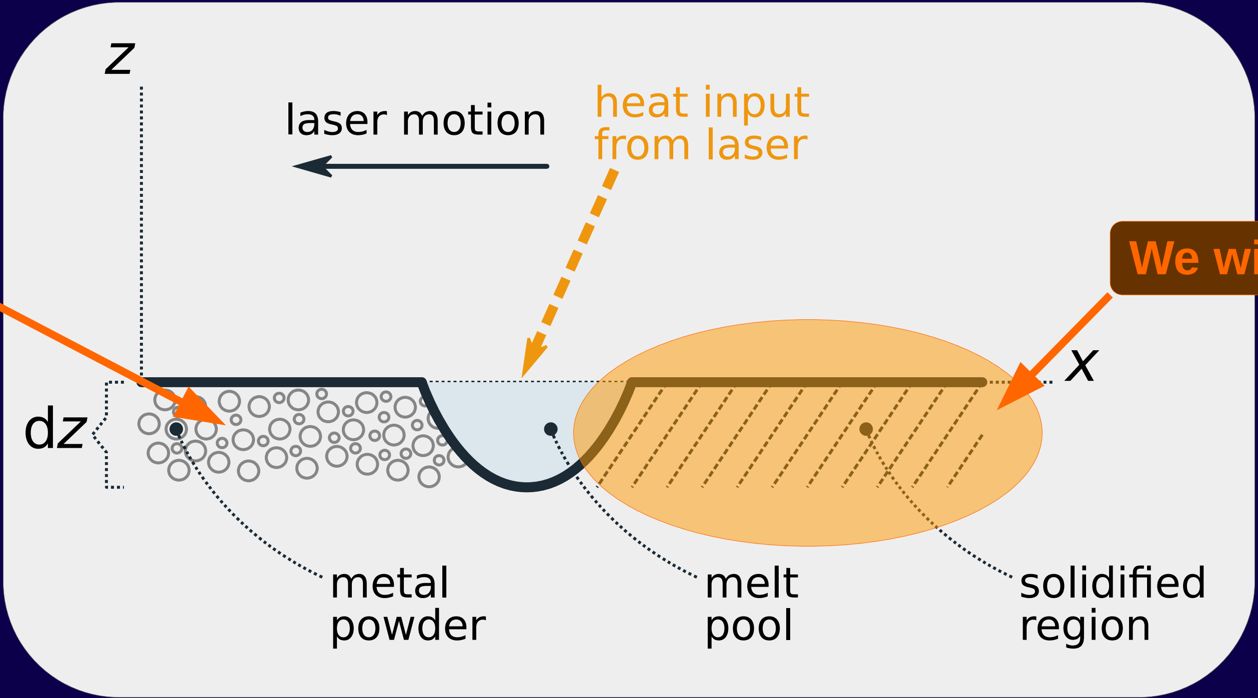
- **Simplify** (to the extreme) **to understand the fundamentals** before tackling more complex cases.
- **Modularize to facilitate easy modification** so various modeling assumptions can be changed at will later.



One layer of a straight line of metal being printed using an LPBF device.

Consider printing metal in a straight line...

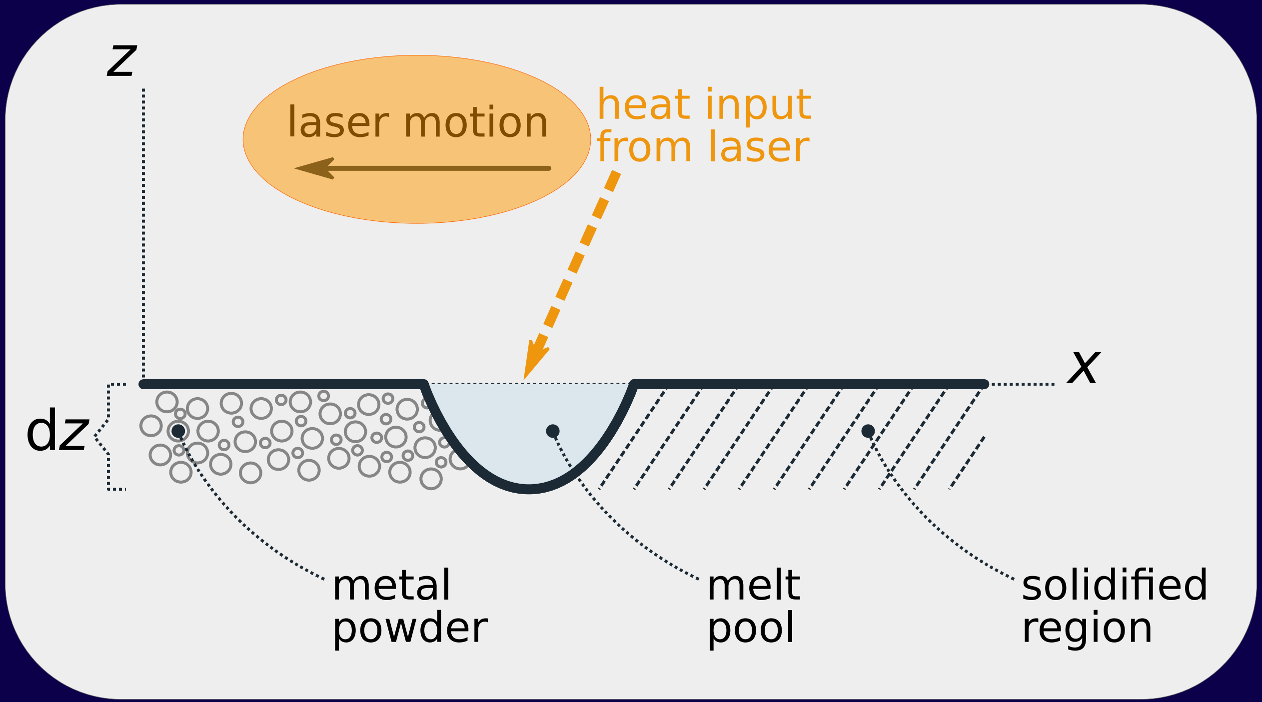
Grains of various sizes
(largest ≈ 20 μm)



We will model this region

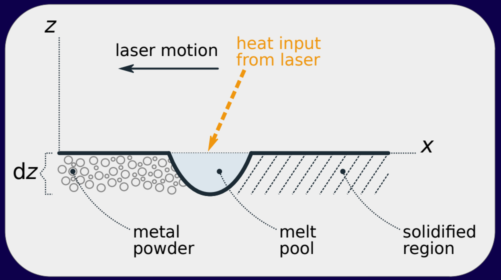
Heat transfer away from the solidified region is
a limiting factor for the manufacturing process.

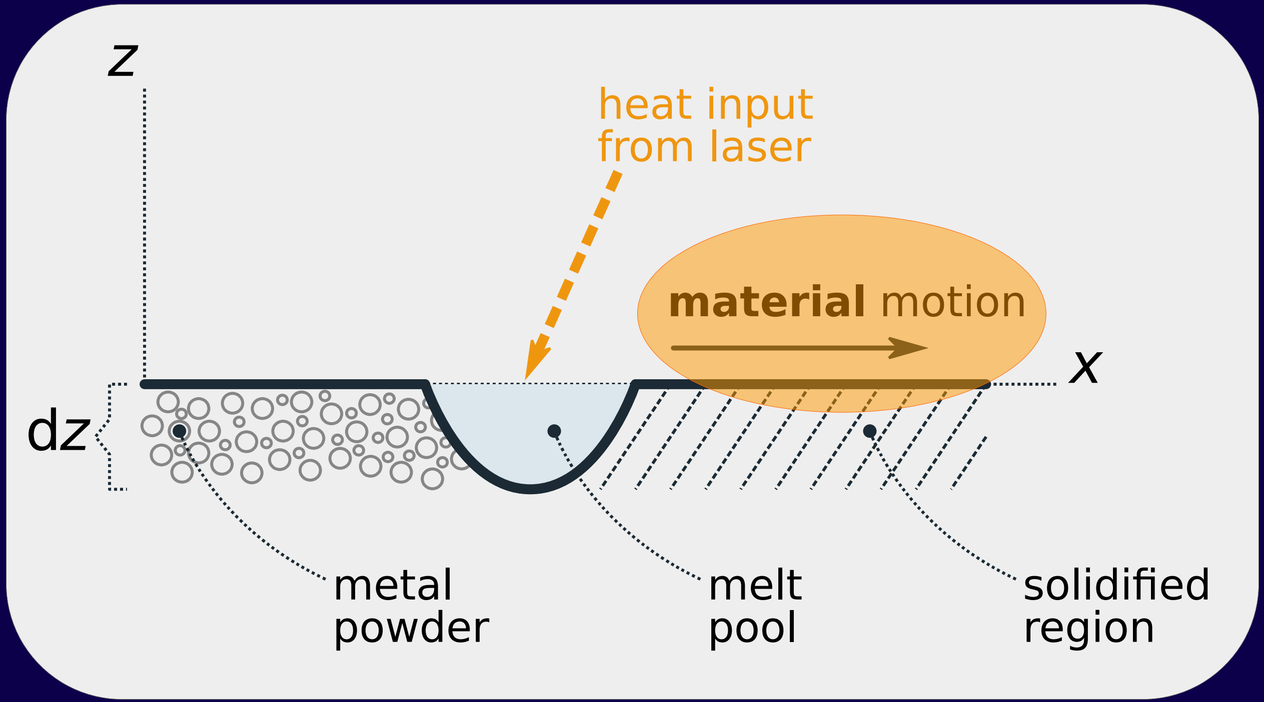
Consider printing metal in a straight line...



Note in this setup, **the laser moves toward the left**.

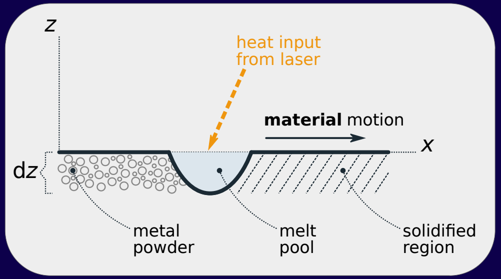
Consider printing metal in a straight line...





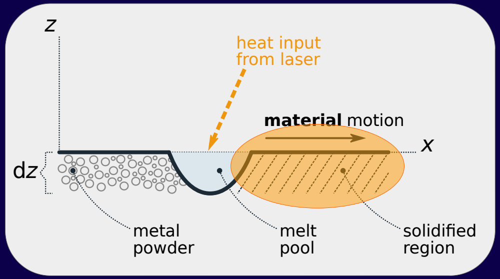
But if we follow the focus spot of the laser, then by Galilean relativity, **the metal moves toward the right**.

Why and how to follow the laser focus spot?



- The **change of perspective** may allow us to see features we would otherwise miss.
  - This allows to easily look at **what happens around the melt pool** as the laser moves.
- Often a **non-stationary steady state** exists for process models like this.
  - To an observer looking at the build cylinder, all quantities vary in time. This requires a transient analysis, and modeling many situational specifics.
  - But to an observer following the focus spot of the laser, some simple configurations, such as the one considered here, admit a state where all quantities stay constant in time (as seen by that observer) even as the printing process proceeds.
- **Simplicity**: fewer parameters; see the core essence of the process being modeled.
  - The considered setup easily extends into 2D (depth, *z* direction) to account for the presence of already printed layers (printing a thin fin).
- **How**: **axially moving materials**
  - Essentially, an Eulerian perspective to solids. Started by Skutch (1897).
  - Applied widely in the process industry, especially in papermaking.
  - For an introduction, see e.g. Banichuk et al. (2020, chapter 5)

Construction of the model



- **Model components**: **balance laws**, **kinematic relation**, **constitutive model**
  - For simplicity, we will model the solidified region only
- The four **fundamental balance laws** of continuum mechanics
  - **Linear momentum** ⇒ Equation of motion (≈ **mechanical model**)

$$\rho \frac{d\boldsymbol{V}}{dt} - \nabla \cdot \boldsymbol{\sigma}^T = \rho \boldsymbol{b}$$

  - **Internal energy** (enthalpy) ⇒ Heat equation for axially moving material (≈ **thermal model**)

$$\rho \frac{de}{dt} = \nabla \cdot \boldsymbol{q} + \boldsymbol{\sigma} : \nabla \boldsymbol{V} + \rho h$$

  - **Angular momentum** ⇒ Cauchy stress tensor **σ** is symmetric (as usual).
  - **Mass** ⇒ No constraints, because trivial axial flow through domain, and density ρ is approximately constant.
- **Kinematic relation**: **small displacements**, for simplicity $\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T]$

Construction of the model

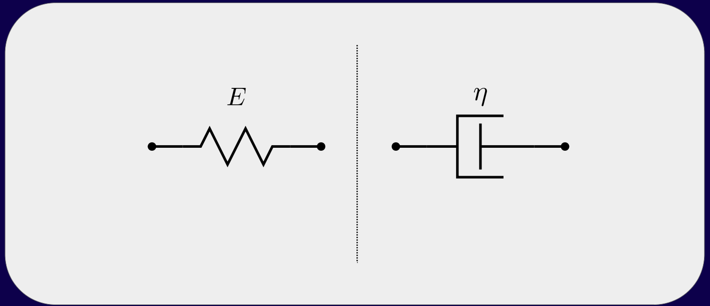
*Only certain kinds of **mechanical** strain create stress.*

1D linear constitutive law

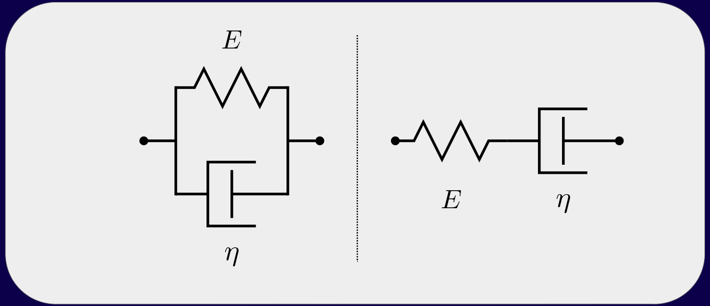
$$[a_0 + a_1 \frac{d}{dt}]\sigma = [b_0 + b_1 \frac{d}{dt}]\varepsilon_{mech}$$

***CAUTION**: **material** derivative.*

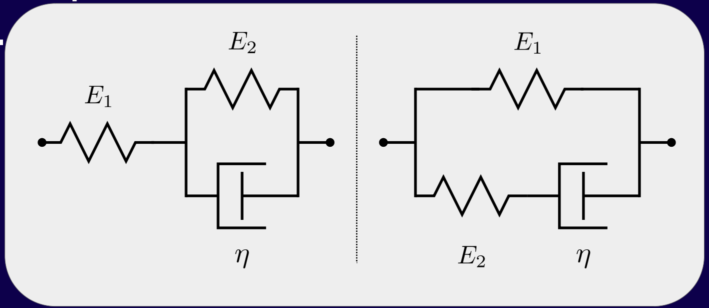
- **Constitutive relation** (stress-strain relation)
  - Material parameters depend on the absolute temperature *T*
  - We restrict to the class of models described by a **linear first-order differential equation**
  - This class includes, notably:
    - **Linear elastic** and **linear viscous** models
    - The **Kelvin-Voigt** and **Maxwell** models
    - The **standard linear solid** (**SLS**).
- We allow any elastic and viscous symmetry groups.
  - **Isotropic**
  - **Transverse isotropic** (layered)
  - **Orthotropic** (layered, plus along laser path vs. perpendicular to it)



Linear elastic solid — Linear viscous fluid

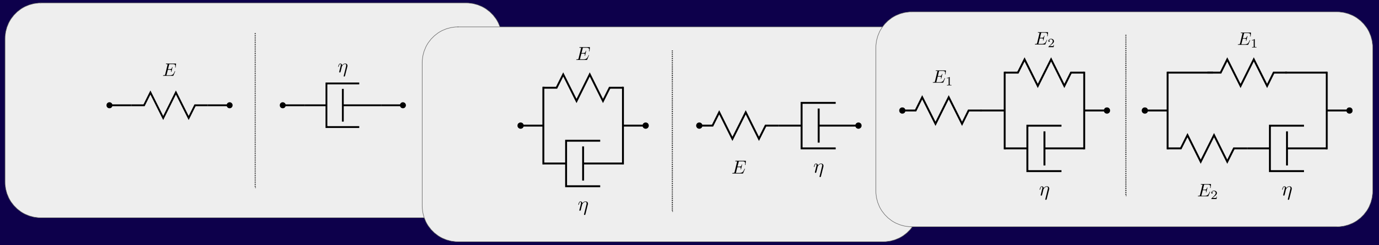


Kelvin–Voigt solid — Maxwell fluid



“Poynting–Thomson” SLS — “Zener” SLS

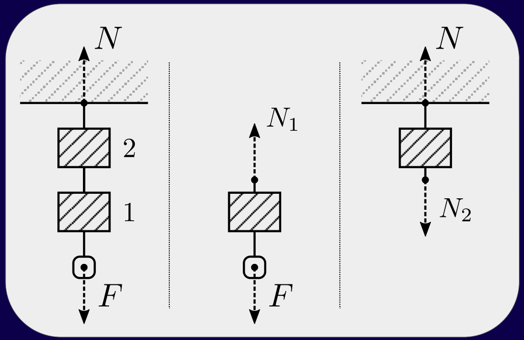
Construction of the model

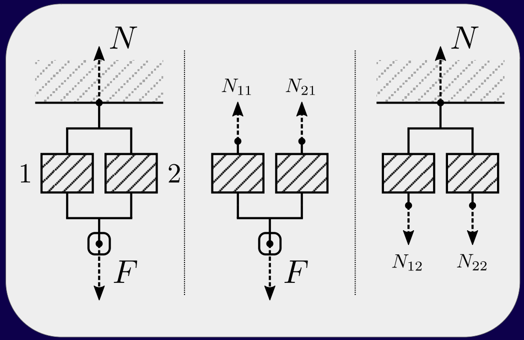


- **Multidimensional networks** are constructed by abstract analogues of the well-known 1D **composition rules**:

$$\varepsilon = \varepsilon_1 + \varepsilon_2, \ \sigma = \sigma_1 = \sigma_2$$

Serial connection





$$\varepsilon = \varepsilon_1 = \varepsilon_2, \ \sigma = \sigma_1 + \sigma_2$$

Parallel connection

- Result (**K**: stiffness, **C**: compliance, **I**$_s$ symmetric rank-4 identity; subscripts *E*, *η* indicate diagram components):

$$I_s : \sigma = K : \varepsilon_{mech}$$

linear elastic

$$I_s : \sigma = K \frac{d}{dt} : \varepsilon_{mech}$$

linear viscous

Starting from these and the composition rules

$$[A_0 + A_1 \frac{d}{dt}] : \sigma = [B_0 + B_1 \frac{d}{dt}] : \varepsilon_{mech}$$

2D and 3D linear constitutive law

Each network component is allowed to have any symmetry group, separately.

$$I_s : \sigma = [K_E + K_\eta \frac{d}{dt}] : \varepsilon_{mech}$$

Kelvin–Voigt

$$[(\frac{\partial C_E}{\partial T}\frac{dT}{dt} + C_\eta) + C_E \frac{d}{dt}] : \sigma = [I_s \frac{d}{dt}] : \varepsilon_{mech}$$

Maxwell

$$[(I_s + K_{E_2} : C_{E_1} + K_\eta : [\frac{\partial C_{E_1}}{\partial T}\frac{dT}{dt}]) + K_\eta : C_{E_1} \frac{d}{dt}] : \sigma = [K_{E_2} + K_\eta \frac{d}{dt}] : \varepsilon_{mech}$$

"Poynting–Thomson" SLS (left side of diagram)

$$A_0 := \frac{\partial C_{E_2}}{\partial T}\frac{dT}{dt} + C_\eta \ , \ [A_0 + C_{E_2} \frac{d}{dt}] : \sigma = [(A_0 : K_{E_1} + C_{E_2} : \frac{\partial K_{E_1}}{\partial T}\frac{dT}{dt}) + (I_s + C_{E_2} : K_{E_1}) \frac{d}{dt}] : \varepsilon_{mech}$$

"Zener" SLS (right side of diagram)

Construction of the model

- **Thermomechanical** coupling
  - **Total strain**:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{mech} + \boldsymbol{\varepsilon}_{th} \quad \Rightarrow \quad \boldsymbol{\varepsilon}_{mech} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{th}$$

$$\left[a_0 + a_1 \frac{d}{dt}\right]\sigma = \left[b_0 + b_1 \frac{d}{dt}\right]\varepsilon_{mech}$$

  where the **thermal strain** describes thermal expansion:

$$\boldsymbol{\varepsilon}_{th} = \boldsymbol{\alpha}\,[T - T_0]$$

  where **α** is the **thermal expansion tensor** (which is part of the **constitutive model**).

- We allow any thermal symmetry group, separately from the elastic and viscous symmetry groups.
  - **Isotropic**
  - **Transverse isotropic**
  - **Orthotropic**

- For example, for a **thermally isotropic** material,

$$\boldsymbol{\alpha} = \alpha\,\mathbf{1}$$

where *α* is the coefficient of thermal expansion (unit 1/K), and **1** is the rank-2 identity tensor.

Governing equations – in multiple dimensions

- Here the equations are presented in strong form; the goal is an exact classical solution for the 1D case.
- **Linear momentum** balance:

(⋆)
$$\rho\left[\frac{\partial^2 \mathbf{u}}{\partial t^2}+2(\mathbf{v}\cdot\nabla)\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{v}\cdot\nabla)(\mathbf{v}\cdot\nabla)\mathbf{u}\right]-\nabla\cdot \mathit{transpose}(\boldsymbol{\sigma})=\rho\mathbf{b}$$

where

(⋆⋆)
$$\begin{aligned}\boldsymbol{\sigma}=&\mathbf{K}_E:\mathit{symm}(\nabla\mathbf{u})-[\mathbf{K}_E:\boldsymbol{\alpha}][T-T_0]\\ &+\mathbf{K}_\eta:\left(\frac{\partial}{\partial t}+(\bar{\mathbf{V}}\cdot\nabla)\right)\mathit{symm}(\nabla\mathbf{u})\\ &-\left[\mathbf{K}_\eta:\left[\frac{\partial\boldsymbol{\alpha}}{\partial T}[T-T_0]+\boldsymbol{\alpha}\right]\right]\left(\frac{\partial}{\partial t}+(\bar{\mathbf{V}}\cdot\nabla)\right)T\end{aligned}$$

Thermoviscoelastic, axially moving, Kelvin–Voigt, with arbitrary symmetry groups.

- **Internal energy** balance:

$$\rho\frac{\partial c}{\partial T}T\left[\frac{\partial T}{\partial t}+(\bar{\mathbf{V}}\cdot\nabla)T\right]+\rho c\left[\frac{\partial T}{\partial t}+(\bar{\mathbf{V}}\cdot\nabla)T\right]-\nabla\cdot(\mathbf{k}\cdot\nabla T)=\boldsymbol{\sigma}:\nabla\bar{\mathbf{V}}+\rho h$$

If $\partial c/\partial T \neq 0$, this is **nonlinear.**

In these equations, $\bar{V}$ is the first-order approximate material parcel velocity in the coordinate frame that follows the laser focus spot:

$$\bar{\mathbf{V}}:=\mathbf{v}+\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{u}$$

To a first approximation, we may replace it by the laser velocity v, which is a constant.

(Actually, we have made that replacement in the inertia terms; these result from approximating the true material parcel acceleration d**V**/dt, as measured in the coordinate frame that follows the laser focus spot.)

Inserting (⋆⋆) into (⋆), final result in index notation:

$$\begin{aligned}&\rho\partial_t^2 u_i+2\rho v_m[\partial_m\partial_t u_i]+\rho[v_m\partial_m][v_n\partial_n]u_i\\ &-\left[[\partial_m T][\partial_T(\mathbf{K}_E)_{milk}]\partial_l u_k+(\mathbf{K}_E)_{imlk}[\partial_m\partial_l u_k]\right]\\ &+[\partial_m T][\partial_T(\mathbf{K}_E)_{mikl}]\alpha_{kl}[T-T_0]+\left[[T-T_0][\partial_T\alpha_{kl}]+\alpha_{kl}\right](\mathbf{K}_E)_{klim}[\partial_m T]\\ &-\left[[\partial_m T][\partial_T(\mathbf{K}_\eta)_{milk}](\partial_t+v_n\partial_n)\partial_l u_k+[\partial_m(\partial_t+v_n\partial_n)\partial_k u_l](\mathbf{K}_\eta)_{klim}\right]\\ &+[\partial_m T][\partial_T(\mathbf{K}_\eta)_{mikl}]\left[[\partial_T\alpha_{kl}][T-T_0]+\alpha_{kl}\right](\partial_t T+\bar{V}_n\partial_n T)\\ &+\left[[T-T_0][\partial_T^2\alpha_{kl}]+2[\partial_T\alpha_{kl}]\right](\mathbf{K}_\eta)_{klim}[\partial_m T](\partial_t T+\bar{V}_n\partial_n T)\\ &+\left[[T-T_0][\partial_T\alpha_{kl}]+\alpha_{kl}\right](\mathbf{K}_\eta)_{klim}\left[[\partial_m\partial_t T]+[\partial_m\bar{V}_n][\partial_n T]+[\bar{V}_n\partial_n][\partial_m T]\right]=\rho b_i\end{aligned}$$

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Governing equations – 1D

- Aim: **solve a simple variant first**, to obtain a point of reference for more complex solutions.
- To do this, we will restrict the previous equations to one dimension.

- With the material parameters allowed to be arbitrary functions of the absolute temperature T, the **equation of motion** is rather complex even in 1D:

- If $\partial c/\partial T \neq 0$, we obtain this **nonlinear heat equation**:

$$\rho\frac{\partial c}{\partial T}T\left[\frac{\partial T}{\partial t}+\bar{V}\frac{\partial T}{\partial x}\right]+\rho c\left[\frac{\partial T}{\partial t}+\bar{V}\frac{\partial T}{\partial x}\right]-\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=\sigma\frac{\partial \bar{V}}{\partial x}+\rho h$$

$$\rho\frac{\partial^2 u}{\partial t^2}+2\rho v\frac{\partial^2 u}{\partial x\,\partial t}+\rho v^2\frac{\partial^2 u}{\partial x^2}$$

inertia

$$-\left[\frac{\partial T}{\partial x}\frac{\partial E}{\partial T}\frac{\partial u}{\partial x}+E\frac{\partial^2 u}{\partial x^2}\right]$$

elastic response

$$+\left[\frac{\partial T}{\partial x}\frac{\partial E}{\partial T}\alpha[T-T_0]\right]+\left[[T-T_0]\frac{\partial \alpha}{\partial T}+\alpha\right]E\frac{\partial T}{\partial x}$$

elastothermal response

$$-\left[\frac{\partial T}{\partial x}\frac{\partial \eta}{\partial x}\left(\frac{\partial^2 u}{\partial x\,\partial t}+v\frac{\partial^2 u}{\partial x^2}\right)+\left(\frac{\partial^3 u}{\partial x^2\,\partial t}+v\frac{\partial^3 u}{\partial x^3}\right)\eta\right]$$

viscous response

$$+\left[\frac{\partial T}{\partial x}\frac{\partial \eta}{\partial x}\left[\frac{\partial \alpha}{\partial T}[T-T_0]+\alpha\right]\left(\frac{\partial T}{\partial t}+\bar{V}\frac{\partial T}{\partial x}\right)\right]$$

$$+\left[[T-T_0]\frac{\partial^2 \alpha}{\partial T^2}+2\frac{\partial \alpha}{\partial T}\right]\eta\frac{\partial T}{\partial x}\left(\frac{\partial T}{\partial t}+\bar{V}\frac{\partial T}{\partial x}\right)$$

$$+\left[[T-T_0]\frac{\partial \alpha}{\partial T}+\alpha\right]\eta\left[\frac{\partial^2 T}{\partial x\,\partial t}+\frac{\partial \bar{V}}{\partial x}\frac{\partial T}{\partial x}+\bar{V}\frac{\partial^2 T}{\partial x^2}\right]=\rho b$$

viscothermal response

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Governing equations – 1D, constant material parameters

- With **constant material parameters**, the 1D governing equations become:

$$\rho\frac{\partial^2 u}{\partial t^2}+2\rho v\frac{\partial^2 u}{\partial x\,\partial t}+[\rho v^2-E]\frac{\partial^2 u}{\partial x^2}-\left(\frac{\partial^3 u}{\partial x^2\,\partial t}+v\frac{\partial^3 u}{\partial x^3}\right)\eta=\rho b-\alpha\underbrace{\left[E\frac{\partial T}{\partial x}+\eta\left[\frac{\partial^2 T}{\partial x\,\partial t}+v\frac{\partial^2 T}{\partial x^2}\right]\right]}_{\text{Effective body force due to thermal expansion.}}$$

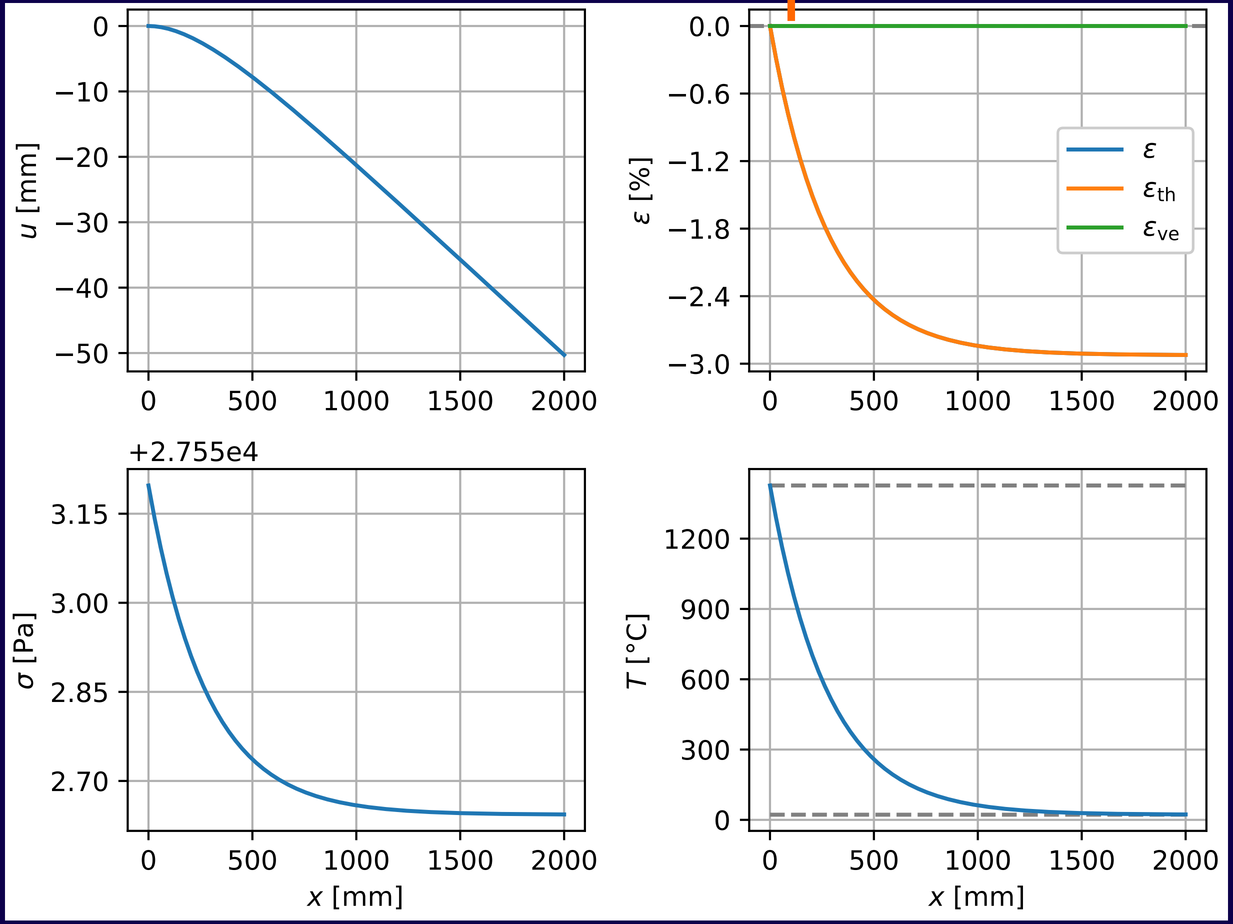
$$\rho c\left[\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial x}\right]-k\frac{\partial^2 T}{\partial x^2}=\rho h$$

- **Third order** in u in the general case, where $\eta v \neq 0$.
  - Axially moving viscous materials typically produce an equation of motion one order higher than in the corresponding classical (not axially moving) case. (E.g. Kurki et al. 2016; Banichuk et al., 2020, chapter 5.)
- 1D; must separately model heat escaping through exposed surface.
  Heat sink, **Newton's law of cooling**: $h(x)=-r[T-T_{ext}]$
- Consider the steady state ($\partial/\partial t \to 0$):
  - For u, introduce auxiliary variable $\varepsilon = \partial u/\partial x$. Then integrate once. This gets rid of two differentiations. Obtain **linear first-order ordinary differential equation**, which has an analytical solution even with arbitrary function coefficients.
  - For T, analytically solvable **linear second-order ordinary differential equation**.
- One-way coupling: T can be solved first.
- Inserting $T(x)$ into the linear momentum balance produces an explicit analytical solution for the displacement $u(x)$.

Numerical results, 316L steel

Axial displacement

Axial strain



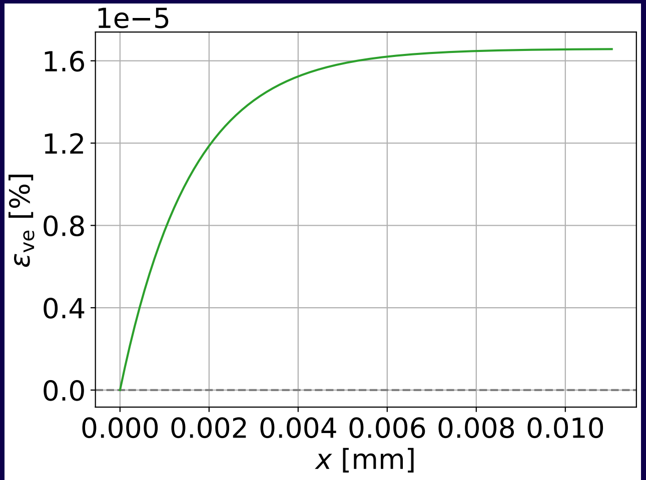
Axial stress

Temperature

total =
thermal +
viscoelastic

melting point

room temperature



Viscoelastic strain. Note the scales.

Almost all of the **total** strain is thermal shrinkage, but only the very small **viscoelastic** contribution produces stress.

Laser scan speed v = 50 mm/s.

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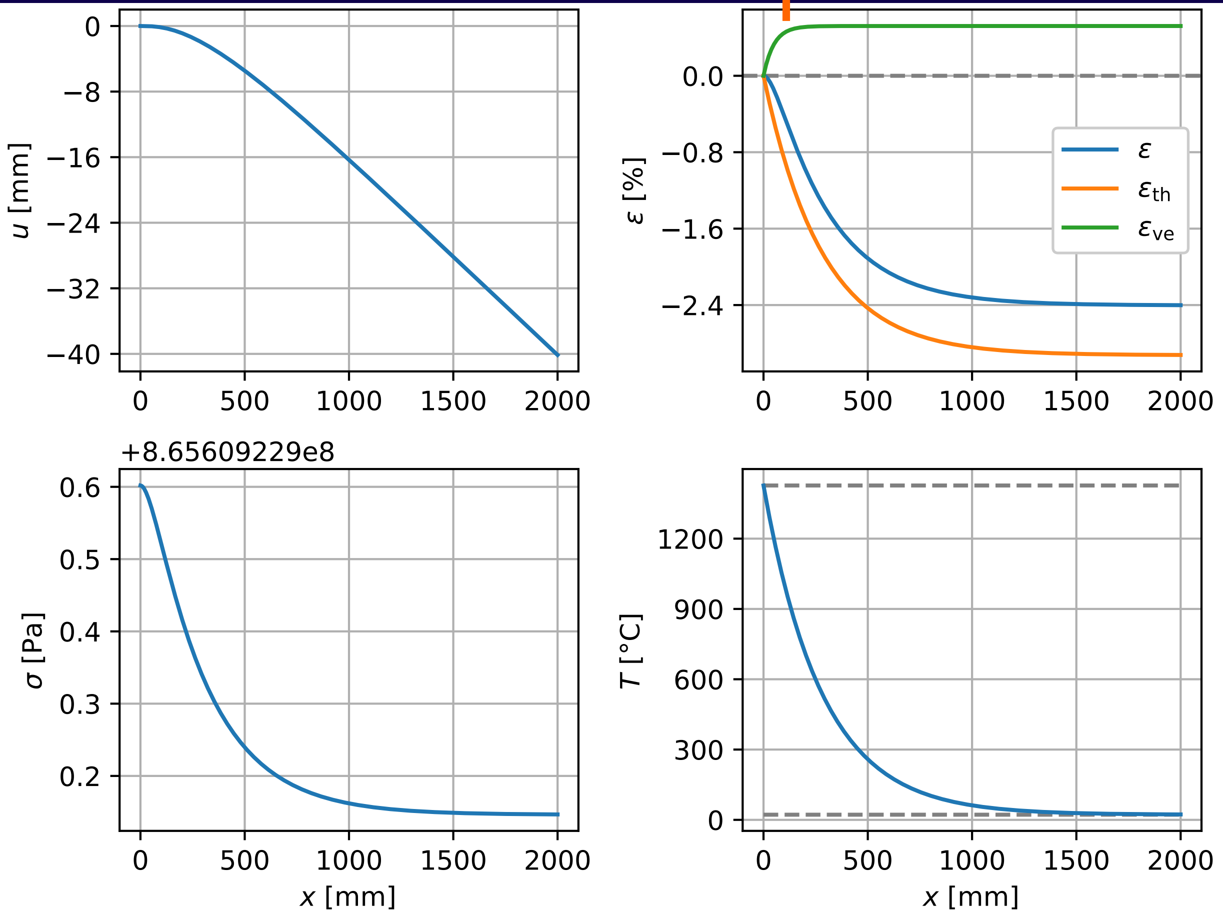
Numerical results, very high viscosity comparison

Axial displacement

Axial strain



Axial stress

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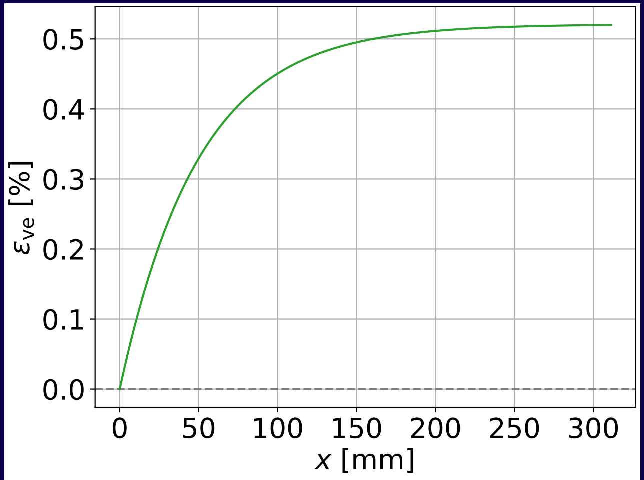

Viscoelastic strain.

Note **high stress level**.

Laser scan speed v = 50 mm/s.

Future plans

- **Near** future:
  - In progress: publish the 1D results in *Journal of Manufacturing and Materials Processing*
  - Solve 2D model (with depth z) numerically in weak form, using finite elements.
    - Standard C^0 continuous elements can be made to work at least with the axially moving Kelvin–Voigt model, using a mixed-form approach (keeping $\boldsymbol{\sigma}$ as a separate variable).
    - *FEniCS* is geared toward agile development of C^0 FEM solvers for custom partial differential equations.

- **Far** future:
  - Numerical simulations using the Standard Linear Solid (SLS) model, for comparison with Kelvin–Voigt.
  - Introduce plastic and/or viscoplastic material models, such as those of the Chaboche family, to account for permanent deformation.

- **Questions to consider**:
  - Role of plastic (permanent) deformation for the quality of the final product?
    - Thermal dependence of material parameters may cause some regions to enter or exit the plastic regime earlier than other regions.
  - Shrinkage during melting due to porosity of the powder?

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Thank you!

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Vipuvoimaa EU:lta 2014–2020

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